Mark Scheme (Results)

Summer 2018

Pearson Edexcel International A Level
In Core Mathematics C34 (WMA02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes..

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1. (i)	$\left\{\int \frac{2x^2+5x+1}{x^2}\,\mathrm{d}x = \int 2+\frac{5}{x}+\frac{1}{x^2}\,\mathrm{d}x\right\}$		
	$= 2x + 5\ln kx - \frac{1}{x}\ \{+c\}$ Where $k \neq 0$ (k is usually 1)	At least one of either $\pm\frac{A}{x} \to \pm\alpha\ln kx$ or $\pm\frac{B}{x^2} \to \pm\beta x^{-1}$; A, B, α, β non zero.	M1
		At least 2 out of the 3 terms are correct. e.g. 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
		$2x + 5\ln kx - \frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1
			[3]
	(i) Alternative by parts I:		
	$\left\{\int (2x^2+5x+1)x^{-2}\,\mathrm{d}x = -\frac{1}{x}(2x^2+5x+1) + \int \frac{1}{x}(4x+5)\,\mathrm{d}x\right\}$		
	$= -2x - 5 - \frac{1}{x} + 4x + 5\ln kx\ \{+c\}$	At least one of either $\pm\frac{A}{x} \to \pm\alpha\ln kx$ or $\pm\frac{B}{x^2} \to \pm\beta x^{-1}$; A, B, α, β non zero.	M1
		At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
	$= 2x - 5 - \frac{1}{x} + 5\ln kx\ \{+c\}$ Where $k \neq 0$ (k is usually 1)	$2x - 5 - \frac{1}{x} + 5\ln kx$ with or without $+c$ Or $2x + 5\ln kx - \frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

	(i) Alternative by parts II:		
	$\left\{\int (2x^2+5x+1)x^{-2}\,\mathrm{d}x = x^{-2}\left(\frac{2x^3}{3}+\frac{5x^2}{2}+x\right)+\int 2x^{-3}\left(\frac{2x^3}{3}+\frac{5x^2}{2}+x\right)\mathrm{d}x\right\}$		
	$= \frac{2x}{3}+\frac{5}{2}+\frac{1}{x}+\frac{4x}{3}+5\ln kx-\frac{2}{x}\{+c\}$	At least one of either $\pm\frac{A}{x}\to\pm\alpha\ln kx$ or $\pm\frac{B}{x^2}\to\pm\beta x^{-1}$; A, B, α, β non zero.	M1
		At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
	$= 2x+\frac{5}{2}-\frac{1}{x}+5\ln kx\ \{+c\}$ Where $k \neq 0$ (k is usually 1)	$2x+\frac{5}{2}-\frac{1}{x}+5\ln kx$ with or without $+c$ or $2x+5\ln kx-\frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

(i) Alternative:		
$\left\{\int \frac{2x^2+5x+1}{x^2}\,\mathrm{d}x = \int 2+\frac{5x+1}{x^2}\,\mathrm{d}x = \int 2+(5x+1)x^{-2}\,\mathrm{d}x\right\} = 2x-\frac{1}{x}(5x+1)+\int \frac{5}{x}\,\mathrm{d}x$		
$= 2x-5-\frac{1}{x}+5\ln kx\ \{+c\}$	At least one of either $\pm\frac{A}{x}\to\pm\alpha\ln kx$ or $\pm\frac{B}{x^2}\to\pm\beta x^{-1}$; A, B, α, β non zero.	M1
	At least 2 out of the 3 terms are correct. At least 2 of $2x, -\frac{1}{x}, 5\ln kx$	A1
	$2x-5-\frac{1}{x}+5\ln kx\ \{+c\}$ with or without $+c$ or $2x+5\ln kx-\frac{1}{x}$ with or without $+c$ all on one line and apply isw once seen. Do not allow $+\frac{1}{-x}$ for $-\frac{1}{x}$	A1

(ii)	$\left\{ \mathrm{I} = \int x\cos 2x\,\mathrm{d}x \right\}$, $\left\{ \begin{array}{l} u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x \Rightarrow v = \frac{1}{2}\sin 2x \end{array} \right\}$		
	$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x \,\{\mathrm{d}x\}$	$\pm\lambda x\sin 2x \pm \mu\int \sin 2x\{\mathrm{d}x\}$ **BUT if the parts formula is quoted incorrectly score M0**	M1
		$\frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,\{\mathrm{d}x\}$ simplified or un-simplified	A1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\ \{+c\}$	$\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ with or without $+c$, $\frac{1}{2}x\sin 2x - \left(-\frac{1}{4}\cos 2x\right)$ is A0	A1
			[3]
			6

	Question 1 Notes	
(i)	**Note**	The 5ln$x$ can appear in different correct forms e.g. 5ln5x or 2.5lnx^2 etc. and allow modulus signs e.g. $5\ln\lvert kx\rvert$
	Note	There are no marks for attempts at $\dfrac{\int 2x^2 + 5x + 1\,\mathrm{d}x}{\int x^2\,\mathrm{d}x}$
(ii)	**Note**	There are no marks for attempts at $\int x\cos x\,\mathrm{d}x$

Question Number	Scheme	Notes	Marks
2.	$x = \frac{3}{2}t - 5, \quad y = 4 - \frac{6}{t}, \quad t \neq 0$		
(a)	$\frac{dx}{dt} = \frac{3}{2}, \quad \frac{dy}{dt} = 6t^{-2}$	**Both** $\frac{dx}{dt} = \frac{3}{2}$ or $\frac{dt}{dx} = \frac{2}{3}$ **and** $\frac{dy}{dt} = 6t^{-2}$ $\frac{dy}{dt}$ can be simplified or un-simplified. **Note:** This mark can be implied.	B1
	So, $\frac{dy}{dx} = \frac{6t^{-2}}{\left(\frac{3}{2}\right)} \left\{= 4t^{-2}\right\}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
	$\left\{\text{When } t = 3, \right\} \frac{dy}{dx} = \frac{4}{9}$	$\frac{4}{9}$	A1 **cao**
			[3]
(b)	• $t = \frac{x+5}{\left(\frac{3}{2}\right)} \Rightarrow y = 4 - \frac{6}{\left(\frac{x+5}{\left(\frac{3}{2}\right)}\right)}$	An attempt to eliminate t.	M1
	• $t = \frac{6}{4-y} \Rightarrow x = \frac{3}{2}\left(\frac{6}{4-y}\right) - 5$ • $\frac{6}{4-y} = \frac{2}{3}(x+5)$	Achieves a correct equation in x and y only.	A1 o.e.
	$\Rightarrow y = 4 - \frac{9}{x+5}$		
	$\Rightarrow y = \frac{4(x+5) - 9}{x+5}$		
	$\Rightarrow y = \frac{4x+11}{x+5}$	$\underline{a = 4}$ and $\underline{b = 11}$ or $\frac{4x+11}{x+5}$	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$, $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
	Alternative 1 for (b):		
	$y = \frac{ax+b}{x+5} \Rightarrow 4 - \frac{6}{t} = \frac{a(1.5t-5)+b}{1.5t-5+5}$		
	$\Rightarrow 4 - \frac{6}{t} = \frac{1.5at - 5a + b}{1.5t} \Rightarrow 6t - 9 = 1.5at - 5a + b$ $\Rightarrow 6t = 1.5at$ or $-9 = -5a + b$	Substitutes for x and y and "compares coefficients" for term in t or constant term	M1
	$a = 4$ **or** b = 11	Correct value for a **or** b	A1
	$a = 4$ **and** b = 11	Correct values for a **and** b	A1
	$x \neq -5$ or $k = -5$	Do not isw so if they have $x \neq -5$, $k \neq -5$ score B0 i.e. penalise contradictory statements.	B1
			[4]
			7

	Alternative 2 for (b):		
	$y=\dfrac{4t-6}{t}=\dfrac{3(4t-6)}{2\frac{3t}{2}}=\dfrac{3(4t-6)}{2(x+5)}=\dfrac{4\times\frac{3t}{2}-9}{(x+5)}=\dfrac{4(x+5)-9}{(x+5)}$ M1: Obtains y in terms of x A1: Correct unsimplified expression		M1A1
	$\Rightarrow\ y=\dfrac{4x+11}{x+5}$	$\underline{a=4}$ and $\underline{b=11}$ or $\dfrac{4x+11}{x+5}$	A1
	$x\neq-5$ or $k=-5$	Do not isw so if they have $x\neq-5$, $k\neq-5$ score B0 i.e. penalise contradictory statements.	B1
			[4]

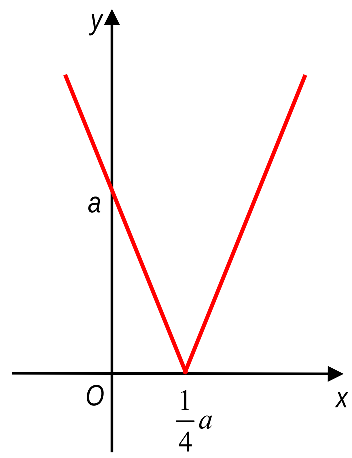
	Question 2 Notes	
2. (a)	**Note**	M1 can also be obtained by substituting $t=3$ into both their $\dfrac{dy}{dt}$ and their $\dfrac{dx}{dt}$ and then dividing their values the correct way round.
	Note	Some candidates may use the Cartesian form in (a) possibly having done (b) first. E.g. $y=\dfrac{4x+11}{x+5}\Rightarrow\dfrac{dy}{dx}=\dfrac{4(x+5)-4x-11}{(x+5)^2}\left(=\dfrac{9}{(x+5)^2}\right)\quad t=3\Rightarrow x=\dfrac{9}{2}-5=-\dfrac{1}{2}$ $\Rightarrow\dfrac{dy}{dx}=\dfrac{9}{\left(-\frac{1}{2}+5\right)^2}=\dfrac{4}{9}$ This would require a complete method to find the Cartesian equation and then B1 for the correct derivative. Then M1 for a complete method attempting the derivative and substituting for x or t and A1 for 4/9 as in the main scheme. The marks for obtaining the Cartesian equation can score in (b) provided their Cartesian equation is seen or used in (b). (i.e. if they do (a) first)

Question Number	Scheme	Notes	Marks
3.	$f(x) = 2^{x-1} - 4 + 1.5x,\ x \in \mathbb{R};\quad x_{n+1} = \frac{1}{3}\left(8 - 2^{x_n}\right),\ x_0 = 1.6$		
(a)	$0 = 2^{x-1} - 4 + 1.5x \Rightarrow 1.5x = 4 - 2^{x-1}$ or $4 - 2^{x-1} = 1.5x$	Sets $f(x) = 0$ and makes $1.5x$ (or kx) the subject of the formula using correct processing so allow sign errors only.	M1
	$\Rightarrow x = \frac{2}{3}\left(4 - 2^{x-1}\right) \Rightarrow x = \frac{1}{3}\left(8 - 2^{x}\right)$ (*) or $\Rightarrow x = \frac{\left(4 - 2^{x-1}\right)}{1.5} \Rightarrow x = \frac{1}{3}\left(8 - 2^{x}\right)$ (*)	$x = \frac{1}{3}\left(8 - 2^{x}\right)$ by **cso** with at least one intermediate step. Do not accept recovery from earlier errors for the A mark. **Note that the "= 0" must be seen at some point for this mark even if only from f(*x*) = 0 at the start.**	A1 *
	Special case: Starts with $1.5x = 4 - 2^{x-1}$ **and completes method with no f(*x*) = 0 is M1A0**		
			[2]
	Alternative working backwards:		
	$x = \frac{1}{3}\left(8 - 2^{x}\right) \Rightarrow 3x = 8 - 2^{x} \Rightarrow 2^{x} - 8 + 3x = 0$ $x - \frac{1}{3}\left(8 - 2^{x}\right) = 0 \Rightarrow 3x - 8 + 2^{x} = 0$	Multiplies by 3 and collects terms to one side or collects terms to one side and multiplies by 3	M1
	$2^{x} - 8 + 3x = 0 \Rightarrow 2^{x-1} - 4 + 1.5x = 0$	Obtains $2^{x-1} - 4 + 1.5x = 0$ by cso.	A1
			[2]
(b)	$x_1 = \frac{1}{3}\left(8 - 2^{1.6}\right)$	For substituting $x_0 = 1.6$ into $\frac{1}{3}\left(8 - 2^{x_0}\right)$. This mark can be implied by x_1 = awrt 1.66	M1
	$x_1 = 1.656,\ x_2 = 1.616$	x_1 = awrt 1.656 **and** x_2 = awrt 1.616	A1
	$x_3 = 1.645$	$x_3 = 1.645$ only **(not awrt)**	A1 **cao**
	Mark their values in the order given i.e. assume their first calculated value is x_1 etc.		
			[3]
(c)	$f(1.6325) = -0.00100095...$ or awrt -1×10^{-3} $f(1.6335) = 0.00157396...$ or awrt 1×10^{-3} or awrt 2×10^{-3}	Chooses a suitable interval for *x*, which is within 1.633 ± 0.0005 and either side of 1.63288… and attempts to evaluate f(*x*) for both values.	M1
	Sign change (negative, positive) (and $f(x)$ is continuous) therefore root $(\alpha = 1.633)$	Both values correct awrt (or truncated) 1 sf, sign change and a conclusion	A1 **cso**
			[2]
			7

		Question 3 Notes
3. (a)	M1	There are other methods for obtaining the printed equation but the M1 scores for setting f(x) = 0 and making kx the subject of the formula using correct processing e.g.
		$\boxed{0 = 2^{x-1} - 4 + 1.5x \Rightarrow \frac{2^x}{2} - 4 + 1.5x = 0 \Rightarrow 3x = 8 - 2^x \text{ M1} \quad \Rightarrow x = \frac{1}{3}\left(8 - 2^x\right) \text{ (*) A1}}$
		$\boxed{0 = 2^{x-1} - 4 + 1.5x \Rightarrow 2^x - 8 + 3x = 0 \Rightarrow 3x = 8 - 2^x \text{ M1} \quad \Rightarrow x = \frac{1}{3}\left(8 - 2^x\right) \text{ (*) A1}}$
3. (c)	A1	**Correct solution only.** Candidate needs to state both of their values for $\mathrm{f}(x)$ to awrt (or truncated) 1sf along with **a reason and conclusion.** Reference to change of sign **or** $\mathrm{f}(1.6325) \times \mathrm{f}(1.6335) < 0$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a conclusion, e.g. $a = 1.633$ (3 dp). Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion could be “change of sign, so true”
	Note	In part (c), candidates can construct their proof using a narrower range than [1.6325, 1.6335] which contains the root 1.632888767

Question Number	Scheme	Notes	Marks
4. (a)	$(1+px)^{-4} = 1 + (-4)(px) + \frac{(-4)(-5)}{2!}(px)^2 + \frac{(-4)(-5)(-6)}{3!}(px)^3 + \ldots$	see notes	M1
	$= 1 - 4px + 10p^2x^2 - 20p^3x^3 + \ldots$ or $= 1 - 4(px) + 10(px)^2 - 20(px)^3 + \ldots$	Three of the four terms correct and simplified.	A1
		All four terms correct and simplified and isw once a correct answer is seen. Must be seen in part (a).	A1
			[3]
(b)	$\left\{ f(x) = \frac{3+4x}{(1+px)^4} = \right\} (3+4x)(1 - 4px + 10p^2x^2 - 20p^3x^3 + \ldots) = \ldots$ Attempts to expand $(3 + 4x) \times$ their part (a) expansion. There should be evidence of at least ($3\times$ one term from part (a)) + ($4x\times$ one term from part (a))		M1
	Note: $f(x) = 3 + (4 - 12p)x + (30p^2 - 16p)x^2 + (40p^2 - 60p^3)x^3 + \ldots$		
	$= 3 \underline{\underline{- 12px + 30p^2x^2}} - 60p^3x^3 \underline{\underline{+ 4x - 16px^2}} + 40p^2x^3$ $\Rightarrow$ $"30p^2 - 16p" = 2"(4 - 12p)"$ **Or** or $2"(30p^2 - 16p)" = "(4 - 12p)"$	**Dependent on the previous M mark** Multiplies out to give exactly two terms in x and exactly 2 terms in x^2 and attempts one coefficient = twice the other. This mark can be implied by later working. Allow x's to be present for this mark	dM1
	$30p^2 - 16p = 2(4 - 12p)$	Correct equation with no x's	A1
	$30p^2 + 8p - 8 = 0$ $\Rightarrow (10p-4)(3p+2) = 0$ or $(5p-2)(6p+4) = 0 \Rightarrow p = \ldots$ or $15p^2 + 4p - 4 = 0 \Rightarrow (5p-2)(3p+2) = 0 \Rightarrow p = \ldots$	**Dependent on the 1st M mark** Correct method for solving a 3TQ leading to at least one value. If working is shown see general guidance for solving 3TQs. If no working is shown then you may need to check to see if their 3TQ solves correctly.	dM1
	$\left\{ p = \frac{2}{5}, -\frac{2}{3} \Rightarrow \text{As } p > 0, \text{ then} \right\} \; p = \frac{2}{5}$	$p = \frac{2}{5}$ only.	A1
			[5]
(c)	$40\left(\frac{2}{5}\right)^2 - 60\left(\frac{2}{5}\right)^3$	Substitutes their $p = \frac{2}{5}$ from part (b) into their coefficient of x^3 (which comes from exactly 2 terms from their expansion)	M1
	Coefficient of x^3 is $\frac{64}{25}$	Allow $\frac{64}{25}$ or $2\frac{14}{25}$. Condone 2.56. Allow $\frac{64}{25}x^3$, $2\frac{14}{25}x^3$, $2.56x^3$ **If 2 answers are offered, score A0**	A1
			[2]
			10

	Question 4 Notes	
4. (a)	**M1**	Uses the binomial expansion with $n = -4$ and $'x' = px$.
	Note	M1 can be given for either $1 + (-4)(px)$ or $\frac{(-4)(-5)}{2!}(px)^2$ or $\frac{(-4)(-5)(-6)}{3!}(px)^3$
(b)	**Note**	Allow recovery in part (b) from missing brackets in part (a). e.g. px^2 now becoming p^2x^2.

Question Number	Scheme	Notes	Marks
5.	$\text{f}: x \to \text{e}^{2x} - 5,\ x \in \mathbb{R}\ ;\ \ \text{g}: x \to \ln(3x-1),\ x \in \mathbb{R},\ x > \frac{1}{3}$		
(i) (a)	$y = \text{e}^{2x} - 5 \Rightarrow x = \text{e}^{2y} - 5$ $x + 5 = \text{e}^{2y} \Rightarrow \ln(x+5) = 2y$	Attempt to make x (or swapped y) the subject using correct processing so allow sign errors only.	M1
	$(y =)\ \frac{1}{2}\ln(x+5)\ \ \left\{(\text{f}^{-1}: x \to)\ \frac{1}{2}\ln(x+5)\right\}$	$\frac{1}{2}\ln(x+5)$ or $\frac{1}{2}\ln\lvert x+5\rvert$ or $\ln(x+5)^{\frac{1}{2}}$. Correct expression ignoring how it is referenced **but must be in terms of x.** Do not allow $\ln(x+5).\frac{1}{2}$ or e.g. $\ln x + 5$ or $\ln(x+5$ unless the correct answer is seen previously or subsequently.	A1
	Domain: $x > -5$ or $(-5, \infty)$	$x > -5$ or $(-5, \infty)$ Condone domain > -5	B1
			[3]
(b)	$\text{fg}(3) = \text{e}^{2\ln(3(3)-1)} - 5$ $\left(\text{NB fg}(x) = 9x^2 - 6x - 4\right)$	g goes into f **and** $x = 3$ is substituted into the result **or** finds $\text{g}(3)\ \{= \ln 8\}$ and substitutes into f	M1
	$\left\{= \text{e}^{2\ln 8} - 5 = 64 - 5\right\} = 59$	59 cao	A1
			[2]
(ii)(a)		A $\bigvee$ shape with the vertex on the positive x-axis (with no significant asymmetry about the vertical through the vertex). The left hand branch must extend into the second quadrant. Do not allow a "y" shape unless the part below the x-axis is dotted or "crossed out"	B1
		States $(0, a)$ and $\left(\frac{1}{4}a, 0\right)$ **or** $\frac{1}{4}a$ marked in the correct position on the x-axis **and** a marked in the correct position on the y-axis. Other points marked on the axes can be ignored.	B1
			[2]
(b)	$\{4x - a = 9a \Rightarrow\}\ \ x = \frac{10a}{4}\ \ \left\{\text{or } x = \frac{5a}{2}\right\}$	$x = \frac{10a}{4}$ or $x = \frac{9a+a}{4}$ or $x = \frac{5a}{2}$ (may be implied)	B1
	$-(4x - a) = 9a$ or $4x - a = -9a$	Attempt at the "second" solution. Accept $-(4x - a) = 9a$ or $4x - a = -9a$ or $-4x = 8\text{a}$. Do not condone (unless recovered) invisible brackets in this case.	M1
	$x = -2a$	$x = -2a$	A1
	$\left\{x = \frac{5}{2}a \Rightarrow\right\}\ \left\lvert\frac{5}{2}a - 6a\right\rvert + 3\left\lvert\frac{5}{2}a\right\rvert;\ = 11a$ $\{x = -2a \Rightarrow\}\ \lvert -2a - 6a\rvert + 3\lvert -2a\rvert;\ = 14a$	Substitutes at least one of their x values from solutions of $\lvert 4x - a\rvert = 9a$ where $x < 6\text{a}$ into $\lvert x - 6a\rvert + 3\lvert x\rvert$ and finds at least one value for $\lvert x - 6a\rvert + 3\lvert x\rvert$ **Must apply the modulus.**	M1
		Both $11a$ and $14a$ and no other answers	A1
			[5]
			12

	Question 5 Notes	
(b)	**Note**	The values of x might be found by squaring: $\lvert 4x-a \rvert = 9a \Rightarrow 16x^2-8ax+a^2=81a^2 \Rightarrow 16x^2-8ax-80a^2=0$ $16x^2-8ax-80a^2=0 \Rightarrow x=\dfrac{5a}{2},\,-2a$ Score as follows: B1 for a correct 3 term quadratic (terms collected after squaring) M1: Solves their 3 term quadratic (usual rules) A1: $x=\dfrac{5a}{2},\,-2a$

Question Number	Scheme	Notes	Mar
6.	$\sqrt{5}\cos q -\ 2\sin q$ ° $R\cos(q + a)$		
(a)	$R = 3$	$R = 3$, **cao** (±3 is B0) ($\sqrt{9}$ is B0)	B1
	$\tan\alpha = \pm\dfrac{2}{\sqrt{5}}$, $\tan\alpha = \pm\dfrac{\sqrt{5}}{2}$ $\Rightarrow$ $\alpha = ...$ (Also allow $\cos\alpha = \pm\dfrac{\sqrt{5}}{3}$ or $\pm\dfrac{2}{3}$, $\sin\alpha = \pm\dfrac{2}{3}$ or $\pm\dfrac{\sqrt{5}}{3}$ $\Rightarrow$ $\alpha = ...$, where "3" is their R.)		M1
	$\alpha = 0.7297276562...$ $\Rightarrow$ $\alpha = 0.7297$ (4 sf)	Anything that rounds to 0.7297 (Degrees is 41.81 and scores A0)	A1
	{**Note**: $\sqrt{5}\cos q -\ 2\sin q = 3\cos(q + 0.7297)$ }		**[3]**
(b)	$\sqrt{5}\cos q -\ 2\sin q = 0.5$		
	$3\cos(\theta + 0.7297) = 0.5$ $\Rightarrow$ $\cos(\theta + 0.7297) = \dfrac{0.5}{3}$	Attempts to use part (a) $"3"\cos(\theta \pm "0.7297") = 0.5$ 9and proceeds to $\cos(\theta \pm "0.7297") = K$, $\lvert K\rvert < 1$ May be implied by $\theta \pm "0.7297" = 1.4033$ or $\theta \pm "0.7297" = \cos^{-1}\left(\dfrac{0.5}{\text{their } 3}\right)(= 1.4033...)$	M1
	$\theta_1 = 0.673648...$ $\Rightarrow$ $\theta_1 = 0.674$ (3 sf)	Anything that rounds to 0.674	A1
	$\theta_2 + "0.7297" = "-1.4033"$ $\Rightarrow$ $\theta_2 = ...$	**dependent on the previous M mark** Correct attempt at a second solution in the range. Usually given for: θ_2 + their 0.7297 = – their 1.4033 $\Rightarrow$ $\theta_2 = ...$	dM1
	$\theta_2 = -2.133048...$ $\Rightarrow$ $\theta_2 = -2.13$ (3 sf)	Anything that rounds to – 2.13	A1
	For solutions in (b) that are otherwise fully correct, if there are extra answers in the range, deduct the final A mark.		
	For candidates who work consistently in degrees in (a) and (b) allow awrt 38.6° and awrt – 122° in part (b) as the A mark will be lost in part (a)		
			[4]
(c)	$\mathrm{f}(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B, \theta \in \mathbb{R};\ -15 \leqslant \mathrm{f}(x) \leqslant 33$ $\Rightarrow$ $-15 \leqslant 3A\cos(\theta + 0.730) + B \leqslant 33$		
	Note that part (c) is now marked as B1M1A1A1		
	$B = 9$	Correct value for B	B1
	$\boxed{\begin{array}{l} 3A + B = 33 \\ -3A + B = -15 \end{array}}$ or $\boxed{\begin{array}{l} 3A + B = -15 \\ -3A + B = 33 \end{array}}$	Writes down at least one pair of simultaneous equations (or inequalities) of the form $\boxed{\begin{array}{l} RA + B = 33 \\ -RA + B = -15 \end{array}}$ or $\boxed{\begin{array}{l} RA + B = -15 \\ -RA + B = 33 \end{array}}$ and finds at least one value for A	M1
	$A = 8$ **or** $A = -8$	One correct value for A	A1
	$A = 8$ **and** $A = -8$	Both values correct	A1
			[4]
			11

(c) **Alt 1**	$B=9$	Correct value for B	B1
	$(2)(A)(3)=33--15$	$(2)(A)(\text{their } R)=33--15 \Rightarrow A=...$	M1
	$A=8$ **or** $A=-8$	One correct value for A	A1
	$A=8$ **and** $A=-8$	Both values correct	A1
			[4]
(c) **Alt 2**	$B=\dfrac{33-15}{2}=9$	Correct value for B	B1
	$3A=33-9 \Rightarrow A=8$	$(\text{their } R)A=33-\text{their } B \Rightarrow A=...$	M1
	$A=8$ **or** $A=-8$	One correct value for A	A1
	$A=8$ **and** $A=-8$	Both values correct	A1
			[4]

	Question 6 Notes	
(c)	**Note**	The M mark may be implied by correct answers so obtaining A = 8 implies M1A1

Question Number	Scheme	Notes	Marks
7.	$V = \frac{1}{3}ph^2(90 - h) = 30ph^2 - \frac{1}{3}ph^3$; $\frac{\mathrm{d}V}{\mathrm{d}t} = 180$		
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 60ph - ph^2$	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} =\right\} \pm ah \pm bh^2$, $a \neq 0, b \neq 0$	M1
		$60ph - ph^2$ Can be simplified or un-simplified.	A1
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow\right\}$ $\left(60ph - ph^2\right)\frac{\mathrm{d}h}{\mathrm{d}t} = 180$ $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} \Rightarrow\right\}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = 180 \times \frac{1}{60\pi h - \pi h^2}$	$\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 180$ or $180 \div$ their $\frac{\mathrm{d}V}{\mathrm{d}h}$ This is for a correct **application** of the chain rule and not for just quoting a correct chain rule.	M1
	When $h = 15$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} =\right\} \frac{1}{60p(15) - p(15)^2} \times 180$ $\left\{= \frac{4}{15p}\right\}$	**Dependent on the previous M mark.** Substitutes $h = 15$ into an expression which is a result of a quotient (or their rearranged quotient) of their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and 180. May be implied by awrt 0.08 or 0.09.	dM1
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = 0.0848826... \Rightarrow\right\}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = \underline{0.085}$ (cm s^{-1}) (2 sf)	**Awrt** 0.085 or allow $\frac{4}{15\pi}$ oe (and isw if necessary)	A1 **cao**
			[5]
			5
	Alternative Method for the first M1A1		
	Product rule: $\left\{\begin{array}{ll} u = \frac{1}{3}ph^2 & v = 90 - h \\ \frac{\mathrm{d}u}{\mathrm{d}h} = \frac{2}{3}ph & \frac{\mathrm{d}v}{\mathrm{d}h} = -1 \end{array}\right\}$		
	$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2}{3}ph(90 - h) + \frac{1}{3}ph^2(-1)$	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} =\right\} \pm\alpha h(90 - h) \pm \beta h^2(-1)$, $\alpha \neq 0, \beta \neq 0$ Can be simplified or un-simplified.	M1
		$\frac{2}{3}ph(90 - h) + \frac{1}{3}ph^2(-1)$ Can be simplified or un-simplified.	A1

	Question 7 Notes	
7.	**Note**	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated for the 1st M1 and/or the 1st A1 but it should be clear that they are differentiating their V.
	Note	$V = \frac{1}{3}\pi h^2(90 - h) \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2}{3}\pi h(90 - h)$ scores M0A0 even though it satisfies the conditions for the derivative.

Question Number	Scheme	Notes	Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},\ l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$; Let $\theta =$ acute angle between PQ and l_1.		
(a)	$\mathbf{i}:\ 1 + \lambda = 6 + \mu$ **(1)** $\mathbf{j}: -3 + 2\lambda = 4 + \mu$ **(2)** $\mathbf{k}:\ 2 + 3\lambda = 1 - \mu$ **(3)**		
	(1) and **(2)** yields $\lambda = 2,\ \mu = -3$ **(1)** and **(3)** yields $\lambda = 1,\ \mu = -4$ **(2)** and **(3)** yields $\lambda = 1.2,\ \mu = -4.6$	Attempts to solve a pair of equations to find at least one of either $\lambda = ...$ or $\mu = ...$	M1
		λ and μ are both correct	A1
	Checking **(3)**: $8 \neq 4$ Checking **(2)**: $-1 \neq 0$ Checking **(1)**: $2.2 \neq 1.4$ l_1 and l_2 do not intersect.	Attempts to show a contradiction	M1
		Correct comparison and a conclusion. Accept "do not meet" and accept "are skew". **Requires all previous work to be correct.**	A1
	Allow a calculation that gives "8 = 4 so the lines do not meet"		
			[4]
	Alternative for part (a):		
	(1) and **(2)** yields $\lambda = 2,\ \mu = -3$ **(1)** and **(3)** yields $\lambda = 1,\ \mu = -4$ **(2)** and **(3)** yields $\lambda = 1.2,\ \mu = -4.6$	Attempts to solve a pair of equations to find at least one of either $\lambda = ...$ or $\mu = ...$	M1
		Shows any two of **(1)** and **(2)** yielding $\lambda = 2$ **(1)** and **(3)** yielding $\lambda = 1$ **(2)** and **(3)** yielding $\lambda = 1.2$ **or** shows any two of **(1)** and **(2)** yielding $\mu = -3$ **(1)** and **(3)** yielding $\mu = -4$ **(2)** and **(3)** yielding $\mu = -4.6$	A1
	E.g. So $2 \neq 1$ l_1 and l_2 do not intersect.	Attempts to show a contradiction	M1
		Correct comparison and a conclusion. Accept "do not meet" and accept "are skew". **Requires all previous work to be correct.**	A1
			[4]

(b)	$\overrightarrow{OP} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix},\ \overrightarrow{OQ} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$		
	$\left(\overrightarrow{PQ} =\right) \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$ or $\left(\overrightarrow{QP} =\right) \begin{pmatrix} -4 \\ -6 \\ 0 \end{pmatrix}$	Full method of finding $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ where P and Q have been found by using $\lambda = 0$ in l_1 and $\mu = -1$ in l_2	M1
		Correct $\overrightarrow{PQ}$ or $\overrightarrow{QP}$. Also allow for direction, $\mathbf{d}_{PQ} = 2\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$ and allow coordinates e.g. (4, 6, 0)	A1
	$\mathbf{d}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},\ \mathbf{d}_{PQ} = \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	M1
	$\cos q = \pm\left(\dfrac{(1)(4)+(2)(6)+(3)(0)}{\sqrt{(1)^2+(2)^2+(3)^2}\,.\sqrt{(4)^2+(6)^2+(0)^2}}\right)$	**Dependent on the previous M mark.** An attempt to apply the dot product formula between $\pm A\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their $\overrightarrow{PQ}$ or $\overrightarrow{QP}$	dM1
	$\cos\theta = \dfrac{16}{\sqrt{14}.\sqrt{52}} \Rightarrow \theta = 53.62985132... = 53.63\ \text{(2 dp)}$	Anything that rounds to 53.63	A1
			[5]

(c)	$\frac{d}{\sqrt{52}} = \sin q$	Writes down a correct trigonometric equation involving the shortest distance, d. e.g. $\frac{d}{\text{their } PQ} = \sin q$, o.e.	M1
	$\left\{d = \sqrt{52}\sin 53.63... \Rightarrow\right\} d = 5.8064... = 5.81\ (3\text{sf})$	Anything that rounds to 5.81	A1
			[2]
	Alternative for part (c): (Let M be the point on l_1 closest to Q)		
	$\overrightarrow{OM} = \begin{pmatrix}1\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} \Rightarrow \overrightarrow{QM} = \begin{pmatrix}1\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}5\\3\\2\end{pmatrix}$ $\begin{pmatrix}\lambda-4\\2\lambda-6\\3\lambda\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix} = 0 \Rightarrow \lambda - 4 + 4\lambda - 12 + 9\lambda = 0$ $\begin{pmatrix}\lambda-4\\2\lambda-6\\3\lambda\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix} = \lambda - 4 + 4\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{8}{7}$ $\lambda = \frac{8}{7} \Rightarrow \overrightarrow{QM} = \frac{1}{7}\begin{pmatrix}-20\\-26\\24\end{pmatrix} \Rightarrow \left\|\overrightarrow{QM}\right\| = \frac{1}{49}\sqrt{20^2 + 26^2 + 24^2}$	Applies a complete and correct method that leads to an expression for the shortest distance	M1
	$= \sqrt{\frac{236}{7}} = 5.81$	Anything that rounds to 5.81	A1
			[2]
			11

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(x)=\dfrac{12}{(2x-1)},\quad 1\leqslant x\leqslant 5;\ y=\dfrac{4}{3}$		
(a)	$\left\{\displaystyle\int \frac{1}{(2x-1)^2}\,\mathrm{d}x\right\} = \dfrac{(2x-1)^{-1}}{(-1)(2)}\ \{+c\}$	$(2x-1)^{-2} \to \pm\lambda(2x-1)^{-1}$ or $\pm\lambda u^{-1}$ where $u=2x\pm1;\ \lambda\neq0$	M1
		$\left(\dfrac{(2x-1)^{-1}}{(-1)(2)}\right)$ or $-\dfrac{1}{2(2x-1)}$ oe with or without $+c$. Can be simplified or un-simplified.	A1
			[2]
(b)	$\pi\displaystyle\int\left(\frac{12}{2x-1}\right)^2\mathrm{d}x$	For $\pi\displaystyle\int\left(\frac{12}{2x-1}\right)^2\mathrm{d}x$ or $\pi\displaystyle\int\frac{144}{(2x-1)^2}\mathrm{d}x$. Ignore limits and $\mathrm{d}x$. Can be implied and the π may be recovered later.	B1
	$V_1 = 144\pi\left[\dfrac{-1}{2(2x-1)}\right]_1^5$		
	$=144(\pi)\left(\left(\dfrac{-1}{2(2(5)-1)}\right)-\left(\dfrac{-1}{2(2(1)-1)}\right)\right)$	Applies x-limits of 5 and 1 to an expression of the form $\pm\beta(2x-1)^{-1};\ \beta\neq0$ and subtracts the correct way round.	M1
		Correct expression for the integrated volume with or without the π. Can be simplified or un-simplified. Can be implied by 64 or 64π.	A1
	$\left\{=-72(\pi)\left(\dfrac{1}{9}-1\right)\ =64(\pi)\right\}$ **Note:** $\pi\displaystyle\int_1^5\left(\frac{12}{2x-1}\right)^2\mathrm{d}x$ or $\displaystyle\int_1^5\left(\frac{12}{2x-1}\right)^2\mathrm{d}x$ evaluated directly as 64π or 64 **<u>with no incorrect working seen</u>** scores M1A1 (presumably on a calculator)		
	$\{V_{\text{cylinder}}\} = \pi\left(\dfrac{4}{3}\right)^2(4)\ \left\{=\dfrac{64}{9}\pi\right\}$	Attempts to use the formula πr^2h with numerical r and h with at least one of $r=\dfrac{4}{3}$ or $h=4$ correct or attempts $\pi\displaystyle\int_1^5\left(\frac{4}{3}\right)^2\mathrm{d}x$ or $\pi\displaystyle\int_0^5\left(\frac{4}{3}\right)^2\mathrm{d}x$	M1
		Correct expression for V_{cylinder} $\pi\left(\dfrac{4}{3}\right)^2(4)$ or $\dfrac{64}{9}\pi$ implies this mark	A1
	$\left\{\mathrm{Vol}(R)=64\pi-\dfrac{64\pi}{9}\right\}\Rightarrow \mathrm{Vol}(R)\ =\ \underline{\dfrac{512}{9}\pi}$	$\dfrac{512}{9}\pi$ or $56\dfrac{8}{9}\pi$	A1
			[6]
			8

		Question 9 Notes
9. (b)	**Note**	See extra notes below for how to mark attempts at $\pi\displaystyle\int_1^5\left(\left(\frac{12}{2x-1}\right)-\left(\frac{4}{3}\right)\right)^2\mathrm{d}x$
	Note	An acceptable approach is $\pi\displaystyle\int_1^5\left(\left(\frac{12}{2x-1}\right)^2-\left(\frac{4}{3}\right)^2\right)\mathrm{d}x$

Attempts at $\pi\int_1^5\left(\left(\frac{12}{2x-1}\right)-\left(\frac{4}{3}\right)\right)^2 \mathrm{d}x$:

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}-\frac{32}{2x-1}+\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $\pi\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}-16\ln(2x-1)+\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}-16\ln 9+\frac{80}{9}\right)-\left(-72+\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{640}{9}\pi-48\ln 9\right)$$

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}+\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $\pi\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}+\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}+\frac{80}{9}\right)-\left(-72+\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{640}{9}\pi\right)$$

$$V=\pi\int_1^5\left(\frac{12}{2x-1}-\frac{4}{3}\right)^2\mathrm{d}x=\pi\int_1^5\left(\frac{144}{(2x-1)^2}-\frac{16}{9}\right)\mathrm{d}x$$

B1 for the embedded $\pi\int\left(\frac{12}{2x-1}\right)^2 \mathrm{d}x$ (π may be recovered later)

$$=\pi\left[-\frac{72}{2x-1}-\frac{16}{9}x\right]_1^5$$

$$=\pi\left[\left(-\frac{72}{9}-\frac{80}{9}\right)-\left(-72-\frac{16}{9}\right)\right]$$

M1A1 for the embedded $-\frac{72}{9}-(-72)$ or $\left(-\frac{72}{9}-(-72)\right)\pi$

$$\left(=\frac{512}{9}\pi\right)$$

Question Number	Scheme	Notes	Marks
10.	$C: x\mathrm{e}^{5-2y} - y = 0$ or $\ln x + 5 - 2y - \ln y = 0$; $P(2\mathrm{e}^{-1}, 2)$ lies on C.		
	Either • $\mathrm{e}^{5-2y} - 2x\mathrm{e}^{5-2y}\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ • $\mathrm{e}^{5-2y} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ • $\frac{1}{x} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ • $\frac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y-5} + 2y\mathrm{e}^{2y-5}$ • $\mathrm{e}^{5} = \mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x}$	Obtains either $\pm A\mathrm{e}^{5-2y} \pm Bx\mathrm{e}^{5-2y}\frac{\mathrm{d}y}{\mathrm{d}x} \pm \frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ **or** $\pm A\mathrm{e}^{5-2y} \pm By\frac{\mathrm{d}y}{\mathrm{d}x} \pm \frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ **or** $\pm\frac{A}{x} \pm K\frac{\mathrm{d}y}{\mathrm{d}x} \pm \frac{B}{y}\frac{\mathrm{d}y}{\mathrm{d}x}(=0)$ **or** $\pm\frac{\mathrm{d}x}{\mathrm{d}y} = \pm A\mathrm{e}^{\pm\alpha\pm 2y} \pm By\mathrm{e}^{\pm\alpha\pm 2y}$ **or** $\pm A\mathrm{e}^{\pm 5} = \pm B\mathrm{e}^{\pm 2y}\frac{\mathrm{d}y}{\mathrm{d}x} \pm Ky\mathrm{e}^{\pm 2y}\frac{\mathrm{d}y}{\mathrm{d}x}$ $A, B, K \neq 0$; a, b can be 0	M1
		Correct differentiation. The " = 0" may be implied by later work.	A1
	Ignore any "$\frac{\mathrm{d}y}{\mathrm{d}x}$ =" in front of their differentiation		
	At P, $\mathrm{e}^{5-2(2)} - 2(2\mathrm{e}^{-1})\mathrm{e}^{5-2(2)}\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\Rightarrow \mathrm{e} - 4\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}}{5}$	Uses $P(2\mathrm{e}^{-1}, 2)$ and their gradient equation to find a numerical value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$. Could have extra or fewer $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms and may have rearranged their expression wrongly before substituting. Accept $\frac{\mathrm{d}y}{\mathrm{d}x}$ = awrt 0.54 as evidence.	M1
	$\left\{m_T = \frac{\mathrm{e}}{5} \Rightarrow\right\}$ • $y - 2 = \frac{\mathrm{e}}{5}\left(x - \frac{2}{\mathrm{e}}\right)$ **or** $x - \frac{2}{\mathrm{e}} = 5\mathrm{e}^{-1}(y - 2)$ • $2 = \frac{\mathrm{e}}{5}(2\mathrm{e}^{-1}) + c \Rightarrow c = \frac{8}{5} \Rightarrow y = \frac{\mathrm{e}}{5}x + \frac{8}{5}$	**Dependent on the previous M mark.** A correct attempt at an equation of the tangent at the point $P(2\mathrm{e}^{-1}, 2)$ using their numerical $\frac{\mathrm{d}y}{\mathrm{d}x}$. If using $y = mx + c$ must reach as far as $c = \ldots$	**d**M1
	$y = 0 \Rightarrow -2 = \frac{\mathrm{e}}{5}\left(x - \frac{2}{\mathrm{e}}\right) \Rightarrow x = -\frac{8}{\mathrm{e}}$ $\left\{\Rightarrow A\left(-\frac{8}{\mathrm{e}}, 0\right)\right\}$ $x = 0 \Rightarrow y - 2 = \frac{\mathrm{e}}{5}\left(-\frac{2}{\mathrm{e}}\right) \Rightarrow y = \frac{8}{5}$ $\left\{\Rightarrow B\left(0, \frac{8}{5}\right)\right\}$	Finds at least one correct intercept. For $-\frac{8}{\mathrm{e}}$, allow awrt -2.94.	A1
	Area $OAB = \frac{1}{2}\left(\frac{8}{\mathrm{e}}\right)\left(\frac{8}{5}\right)$	**Dependent on both previous M marks.** Applies $\frac{1}{2}$(their x_A)(their y_B) where their x_A and y_B are **exact**. Condone a method that gives a negative area.	**dd**M1
	$= \frac{32}{5\mathrm{e}}$ or $\frac{32}{5}\mathrm{e}^{-1}$	$\frac{32}{5\mathrm{e}}$ or $\frac{32}{5}\mathrm{e}^{-1}$. Allow $6.4\mathrm{e}^{-1}$ but not e.g. $\frac{64}{10\mathrm{e}}$	A1
			[7]
			7
	Question 10 Notes		
	Note	Accept the alternative notation for the differentiation e.g. $\mathrm{e}^{5-2y}\mathrm{d}x - 2x\mathrm{e}^{5-2y}\mathrm{d}y - \mathrm{d}y = 0$	

	Note	The 2nd and 3rd method marks are available for work in decimals but the final method mark requires exact work.
	Note	Accept y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$

Question Number	Scheme	Notes	Marks
11. (a)	$x = 3\sec\theta = \frac{3}{\cos\theta} = 3(\cos\theta)^{-1}$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3(\cos\theta)^{-2}(-\sin\theta)$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \pm k\left((\cos\theta)^{-2}(\sin\theta)\right)$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)}}(\tan\theta) = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3\tan\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$	Convincing proof with no notational or other errors such as missing θ's or missing signs or inconsistent variables. But use of $\cos^{-1}\theta$ as $\frac{1}{\cos\theta}$ is OK. Must see both $\underline{\underline{\text{underlined steps}}}$. Allow $3\tan\theta\sec\theta$	A1 *
	If the $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ is included on the lhs it must be correct but condone its omission and apply isw if possible if it appears correctly at some point in their working.		
			[2]
(a) **Alt 1**	$x = 3\sec\theta = \frac{3}{\cos\theta}$		
	$\left\{\begin{matrix} u = 3 & v = \cos\theta \\ \frac{\mathrm{d}u}{\mathrm{d}\theta} = 0 & \frac{\mathrm{d}v}{\mathrm{d}\theta} = -\sin\theta \end{matrix}\right\}$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{0(\cos\theta) - (3)(-\sin\theta)}{(\cos\theta)^2}$	Accept $\frac{0\times(\cos\theta) \pm (3)(\sin\theta)}{(\cos\theta)^2}$ as evidence but if the quotient rule is quoted, it must be correct.	M1
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}} = \underline{\underline{3\sec\theta\tan\theta}}$ * Or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \left\{\frac{3\sin\theta}{\cos^2\theta}\right\} = \underline{\underline{\left(\frac{3}{\cos\theta}\right)}}(\tan\theta) = \underline{\underline{3\sec\theta\tan\theta}}$ *	Convincing proof with no notational or other errors such as missing θ's. Must see both $\underline{\underline{\text{underlined steps}}}$. Allow $3\tan\theta\sec\theta$	A1 *
	If the $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ is included on the lhs it must be correct but condone its omission and apply isw if possible if it appears correctly at some point in their working.		
			[2]

(b)	$y = \dfrac{\sqrt{x^2-9}}{x}, x \geqslant 3;\ \ x = 3\sec\theta \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 3\sec\theta\tan\theta$		
	$\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,\mathrm{d}x = \int \frac{\sqrt{((3\sec\theta)^2-9)}}{3\sec\theta}\,3\sec\theta\tan\theta\,\mathrm{d}\theta$	Full substitution of $\dfrac{\sqrt{x^2-9}}{x}$ in terms of θ and "dx" as their "$\pm k\sec\theta\tan\theta$". This may be implied if they reach $\pm\lambda\int \tan^2\theta\{\mathrm{d}\theta\}$ with no incorrect working seen.	M1
	Note: If $\sqrt{x^2-9}$ is simplified incorrectly to $x-3$ the first mark is still available for a full substitution. (Any subsequent marks are unlikely)		
	$= 3\displaystyle\int \tan^2\theta\,\mathrm{d}\theta$	$\pm\lambda\int \tan^2\theta\{\mathrm{d}\theta\}$ (Allow $\pm\lambda\int \tan\theta\tan\theta\{\mathrm{d}\theta\}$)	M1
		$3\int \tan^2\theta\{\mathrm{d}\theta\}$ (Allow $3\int \tan\theta\tan\theta\{\mathrm{d}\theta\}$)	A1
	$= (3)\displaystyle\int (\sec^2\theta - 1)\,\mathrm{d}\theta$	**Dependent on the previous M mark** applies $\tan^2\theta = \sec^2\theta - 1$	dM1
	$= (3)(\tan\theta - \theta)$	$k\tan^2\theta \rightarrow k(\tan\theta - \theta)$	A1
	$\left\{\text{Area}(R) = \displaystyle\int_3^6 \frac{\sqrt{(x^2-9)}}{x}\,\mathrm{d}x = \left[3\tan\theta - 3\theta\right]_0^{\frac{\pi}{3}}\right\}$		
	$= \left\{3\tan\left(\dfrac{\pi}{3}\right) - 3\left(\dfrac{\pi}{3}\right)\right\} - \{0\}$	Substitutes limits of $\dfrac{\pi}{3}$ and 0 into an expression that contains a trigonometric and an algebraic function and subtracts the correct way round. [**Note:** Limit of 0 can be implied.] If they return to x, they must substitute the limits 6 and 3 and subtract the correct way round having previously obtained a trigonometric and an algebraic function.	M1
	$= 3\sqrt{3} - \pi$	$3\sqrt{3} - \pi$	A1
	$\left[3\tan\theta - 3\theta\right]_0^{\frac{\pi}{3}} = 3\sqrt{3} - \pi$ can score the final M1A1 but if no substitution is shown and the answer is incorrect, score M0		
			[7]
			9
	Question 11 Notes		
11. (a)	**Note**	$x = \dfrac{3}{\cos\theta} \Rightarrow x\cos\theta = 3 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta}\cos\theta - x\sin\theta = 0 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{x\sin\theta}{\cos\theta} = 3\sec\theta\tan\theta$ is M1A1. M1 for $\pm A\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\cos\theta \pm B\,x\sin\theta = 0$	
(b)	**Note**	A decimal answer of 2.054559769… (without a correct **exact** answer) is A0.	

Question Number	Scheme	Notes	Marks
12.	$\cot x - \tan x \equiv 2\cot 2x$		
(a)	$\cot x - \tan x = \dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}$	Attempts to write both $\cot x$ and $\tan x$ in terms of $\sin x$ and $\cos x$ only	M1
	$= \dfrac{\cos^2 x}{\sin x \cos x} - \dfrac{\sin^2 x}{\cos x \sin x} \left(= \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} \right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$= \dfrac{\cos 2x}{\frac{1}{2}\sin 2x} \left(= \dfrac{2\cos 2x}{\sin 2x} \right)$	**Dependent on both the previous M marks.** Evidence of correctly applying either $\cos 2x = \cos^2 x - \sin^2 x$ or $\sin 2x = 2\sin x \cos x$	ddM1
	$= 2\cot 2x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]
(a) **Alt 1**	$\cot x - \tan x = \dfrac{1}{\tan x} - \tan x$	Writes $\cot x$ in terms of $\tan x$	M1
	$\dfrac{1}{\tan x} - \dfrac{\tan^2 x}{\tan x} \left(= \dfrac{1 - \tan^2 x}{\tan x} \right)$	**Dependent on the previous M mark** Attempts to find the same denominator for both fractions	dM1
	$\dfrac{2}{\tan 2x}$	**Dependent on both the previous M marks.** Evidence of correctly applying $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	ddM1
	$= 2\cot 2x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1*
			[4]
(a) **Alt 2**	$2\cot 2x = \dfrac{2}{\tan 2x}$	Applies $\cot 2x = \dfrac{1}{\tan 2x}$	M1
	$= \dfrac{2}{\frac{2\tan x}{1 - \tan^2 x}}$	**Dependent on the previous M mark** Attempts to apply the double angle formula for $\tan 2x$	dM1
	$= \dfrac{1 - \tan^2 x}{\tan x} = \dfrac{1}{\tan x} - \tan x$	**Dependent on both the previous M marks.** Obtains a rational fraction with a single denominator and attempts to split this up into 2 terms	ddM1
	$= \cot x - \tan x$ (*)	Correct proof with no notational or other errors such as missing x's or inconsistent variables.	A1 *
			[4]

(b)	$5+\cot(\theta-15°)-\tan(\theta-15°)=0$		
	$\Rightarrow 5+2\cot(...)=0$	Obtains an equation of this form.	M1
	$\cot(...)=-\frac{5}{2} \Rightarrow \tan(...)=-\frac{2}{5}$	Obtains an equation of the form $\tan(...)=\pm\frac{2}{5}$	M1
	$2\theta-30=\tan^{-1}\left(-\frac{2}{5}\right)$	Can be implied by e.g. $2\theta-30=\text{awrt}-21.8$ or $2\theta-30=\text{awrt } 158.2$	A1
	$\theta=\text{awrt } 4.1°$ **or** $\theta=\text{awrt } 94.1°$	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta=\text{awrt } 4.1°$ **and** $\theta=\text{awrt } 94.1°$	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
	Alternative to part (b):		
	$5+\cot(...)-\tan(...)=0 \Rightarrow 5\tan(...)+1-\tan^2(...)$ $\tan^2(...)-5\tan(...)-1=0$ Multiples through by $\tan(...)$ to obtain a 3TQ in $\tan(...)$		M1
	$\tan(...)=\frac{5\pm\sqrt{25+4}}{2}$	Solves their 3TQ and proceeds to tan(…) =	M1
	$(\theta-15°)=\tan^{-1}\left(\frac{5\pm\sqrt{25+4}}{2}\right)$	Can be implied by e.g. $\theta-15=79.099...$ or $\theta-15=-10.900...$	A1
	$\theta=\text{awrt } 4.1°$ **or** $\theta=\text{awrt } 94.1°$	One correct answer e.g. anything that rounds to 4.1 or anything that rounds to 94.1	A1
	$\theta=\text{awrt } 4.1°$ **and** $\theta=\text{awrt } 94.1°$	Both answers correct. Ignore any extra answers out of range but withhold this mark if there are any extra values in range.	A1
			[5]
			9

Question 12 Notes

(a)	**Note**	Allow candidates to "meet in the middle" e.g. $\text{lhs}=\frac{1}{\tan x}-\tan x=\frac{1-\tan^2 x}{\tan x}$: M1dM1 as in Alt1 $\text{rhs}=2\cot 2x=\frac{2}{\tan 2x}=\frac{2}{\frac{2\tan x}{1-\tan^2 x}}$: ddM1 uses double angle for tan2x on rhs $=\frac{1-\tan^2 x}{\tan x}$ so lhs = rhs A1 Correct proof with conclusion

Question Number	Scheme	Notes	Marks
13. (a)	$\dfrac{1}{(4-x)(2-x)} = \dfrac{A}{(4-x)} + \dfrac{B}{(2-x)}$ $\Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \ldots$ or $B = \ldots$	Forming a correct identity. For example, $1 \equiv A(2-x) + B(4-x)$ from $\dfrac{1}{(4-x)(2-x)} = \dfrac{A}{(4-x)} + \dfrac{B}{(2-x)}$ and finds at least one of $A = \ldots$ or $B = \ldots.$	M1
	$A = -\dfrac{1}{2},\ B = \dfrac{1}{2}$ giving $\dfrac{-\frac{1}{2}}{(4-x)} + \dfrac{\frac{1}{2}}{(2-x)}$	$\dfrac{-\frac{1}{2}}{(4-x)} + \dfrac{\frac{1}{2}}{(2-x)}$ or any equivalent form. ***Cannot*** be recovered from part (b) and must be stated as **partial fractions** in (a) **and not just the values of the constants**.	A1
	Correct answer in (a) scores both marks		
			[2]
(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2-x),\ t \geqslant 0$		
	$\displaystyle\int \frac{1}{(4-x)(2-x)}\,\mathrm{d}x = \int k\,\mathrm{d}t$	Separates variables correctly. $\mathrm{d}x$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.	B1 oe
	$\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt\ \ (+c)$ Or e.g. $\dfrac{1}{2}\ln(8-2x) - \dfrac{1}{2}\ln(4-2x) = kt\ \ (+c)$	$\pm\lambda\ln\alpha(4-x) \pm \mu\ln\beta(2-x)$, $\lambda \neq 0,\ \mu \neq 0,\ \alpha \neq 0,\ \beta \neq 0$	M1
		$\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt$ oe Do not condone missing brackets around the $4-x$ and/or the $2-x$ unless they are implied by subsequent work.	A1
	$\{t = 0,\ x = 0 \Rightarrow\}\quad \dfrac{1}{2}\ln 4 - \dfrac{1}{2}\ln 2 = 0 + c\ \left\{\Rightarrow c = \dfrac{1}{2}\ln 2\right\}$	Using both $t = 0$ and $x = 0$ in an integrated equation containing a constant of integration.	M1
	$\dfrac{1}{2}\ln(4-x) - \dfrac{1}{2}\ln(2-x) = kt + \dfrac{1}{2}\ln 2 \Rightarrow \ln\left(\dfrac{(4-x)}{2(2-x)}\right) = 2kt$		
	$\dfrac{4-x}{4-2x} = \mathrm{e}^{2kt}$	Starting from an equation of the form $\pm\lambda\ln(a-x) \pm \mu\ln(b-x) = \pm kt + c,\ \lambda, \mu, \alpha, \beta \neq 0$, and applies a fully correct method to eliminate their logarithms. (Sign errors only). **Must have a constant of integration that need not be evaluated.**	M1
	$4 - x = 4\mathrm{e}^{2kt} - 2x\mathrm{e}^{2kt} \Rightarrow 4 - 4\mathrm{e}^{2kt} = x - 2x\mathrm{e}^{2kt}$ $\Rightarrow 4 - 4\mathrm{e}^{2kt} = x(1 - 2\mathrm{e}^{2kt}) \Rightarrow x = \dfrac{4 - 4\mathrm{e}^{2kt}}{1 - 2\mathrm{e}^{2kt}}$ (*)	**Dependent on the previous M mark** A complete correct method of rearranging to make x the subject allowing sign errors only. **Must have a constant of integration that need not be evaluated.**	dM1
		Achieves the given answer with no errors.	A1 *
			[7]

(c)	$\left\{\dfrac{4-x}{4-2x} = e^{2kt}\right\} \Rightarrow e^{2kt} = \dfrac{4-1}{4-2}\ \left\{=\dfrac{3}{2}\right\}$	Substitutes $x = 1$ leading to e^{2kt} = value **Note:** $k = 0.1$	M1
	$t = \dfrac{1}{2(0.1)}\ln\left(\dfrac{3}{2}\right) = 2.027325541...\ \{= 2.03 \text{ (s) } (3 \text{ sf})\}$	Anything that rounds to 2.03 Do not apply isw here and do not accept the exact value.	A1
			[2]
			11

	Question 13 Notes	
(c)	**Note**	May use an earlier form of their equation to find t when $x = 1$ e.g. $\dfrac{1}{2}\ln(3) - \dfrac{1}{2}\ln(1) = 0.1t + \dfrac{1}{2}\ln 2 \Rightarrow 0.2t = \ln\dfrac{3}{2}$ M1: For correct processing leading to kt = value $t = \dfrac{1}{2(0.1)}\ln\left(\dfrac{3}{2}\right) = 2.027325541...\ \{= 2.03 \text{ (s) } (3 \text{ sf})\}$ A1: Anything that rounds to 2.03 Do not apply isw here

Question Number	Scheme	Notes	Marks
14.	(a) $y=\dfrac{(x^2-4)^{\frac{1}{2}}}{x^3},\ x>2$; (b) $\mathrm{f}(x)=\dfrac{24(x^2-4)^{\frac{1}{2}}}{x^3},\ x>2$		
(a)	$u=(x^2-4)^{\frac{1}{2}}$ $\quad v=x^3$	$(x^2-4)^{\frac{1}{2}}\to\pm\lambda x(x^2-4)^{-\frac{1}{2}},\ \lambda\neq 0.$ **Can be implied.**	M1
	$\dfrac{du}{dx}=\dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ $\quad\dfrac{dv}{dx}=3x^2$	$(x^2-4)^{\frac{1}{2}}\to\dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ un-simplified or simplified. **Can be implied.**	A1
	$\dfrac{dy}{dx}=\dfrac{\frac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}(x^3)-3x^2(x^2-4)^{\frac{1}{2}}}{(x^3)^2}$	Applies $\dfrac{vu'-uv'}{v^2}$ with $u=(x^2-4)^{\frac{1}{2}},\ v=x^3$, their u' and their v'	M1
		Correct $\dfrac{dy}{dx}$, un-simplified or simplified.	A1
	$=\dfrac{x^4(x^2-4)^{-\frac{1}{2}}-3x^2(x^2-4)^{\frac{1}{2}}}{x^6}$		
	Either • $\dfrac{dy}{dx}=\dfrac{(x^2-4)^{-\frac{1}{2}}(x^4-3x^2(x^2-4))}{x^6}$ **or** • $\dfrac{dy}{dx}=\dfrac{x^2(x^2-4)^{-\frac{1}{2}}-3(x^2-4)^{\frac{1}{2}}}{x^4}$	Simplifies $\dfrac{dy}{dx}$ by **either** correctly taking out a factor of $(x^2-4)^{-\frac{1}{2}}$ from their numerator **or** by multiplying numerator and denominator by $(x^2-4)^{\frac{1}{2}}$	M1
	$\dfrac{dy}{dx}=\dfrac{x^2-3(x^2-4)}{x^4(x^2-4)^{\frac{1}{2}}}\Rightarrow\dfrac{dy}{dx}=\dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$	Correct algebra leading to $\dfrac{dy}{dx}=\dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$ $\{A=-2\}$	A1
			[6]
	Alternative by product rule:		
	$u=(x^2-4)^{\frac{1}{2}}$ $\quad v=x^{-3}$	$(x^2-4)^{\frac{1}{2}}\to\pm\lambda x(x^2-4)^{-\frac{1}{2}},\ \lambda\neq 0.$ **Can be implied.**	M1
	$\dfrac{du}{dx}=\dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ $\quad\dfrac{dv}{dx}=-3x^{-4}$	$(x^2-4)^{\frac{1}{2}}\to\dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}$ un-simplified or simplified. **Can be implied.**	A1
	$\dfrac{dy}{dx}=\dfrac{1}{2}(2x)(x^2-4)^{-\frac{1}{2}}(x^{-3})+\left(-3x^{-4}\right)(x^2-4)^{\frac{1}{2}}$	Applies $vu'+uv'$ with $u=(x^2-4)^{\frac{1}{2}},\ v=x^{-3}$, their u' and their v'	M1
		Correct $\dfrac{dy}{dx}$, un-simplified or simplified.	A1
	$\dfrac{dy}{dx}=\dfrac{1}{x^2(x^2-4)^{\frac{1}{2}}}-\dfrac{3(x^2-4)^{\frac{1}{2}}}{x^4}=\ldots$	Simplifies $\dfrac{dy}{dx}$ by correctly writing as two fractions and attempts a common denominator	M1
	$\dfrac{dy}{dx}=\dfrac{x^2-3(x^2-4)}{x^4(x^2-4)^{\frac{1}{2}}}\Rightarrow\dfrac{dy}{dx}=\dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$	Correct algebra leading to $\dfrac{dy}{dx}=\dfrac{-2x^2+12}{x^4(x^2-4)^{\frac{1}{2}}}$ $\{A=-2\}$	A1
			[6]

(b)	$\left\{ f'(x) = \dfrac{24(-2x^2+12)}{x^4(x^2-4)^{\frac{1}{2}}} = 0 \Rightarrow \right\}$ $24(-2x^2+12)=0 \Rightarrow x^2 = 6$	Sets the numerator of their $\dfrac{dy}{dx}=0$ or the numerator of their $f'(x)=0$ and solves to give $x^2 = K$, where $K>0$	M1
	$\Rightarrow x=\sqrt{6}$ or awrt 2.45	$x=\sqrt{6}$ or awrt 2.45 (Allow $x=\pm\sqrt{6}$ or awrt $\pm$ 2.45) (may be implied by their working)	A1
	$f\left(\sqrt{6}\right) = \dfrac{24(6-4)^{\frac{1}{2}}}{\left(\sqrt{6}\right)^3}; = \dfrac{24\sqrt{2}}{6\sqrt{6}} = \dfrac{4}{\sqrt{3}}$ or $\dfrac{4}{3}\sqrt{3}$	**Dependent on the previous M mark.** Substitutes their found x into $f(x)$ or the given expression from part (a). May be implied by awrt 2.3 or may need to check their value.	dM1
		cso leading to $f_{max} = \dfrac{24\sqrt{2}}{6\sqrt{6}}$ or $\dfrac{4}{\sqrt{3}}$ or $\dfrac{4}{3}\sqrt{3}$ **(Must be exact here)**	A1
	Range: $0 < f(x) \leqslant \dfrac{4}{3}\sqrt{3}$ or $0 < y \leqslant \dfrac{4}{\sqrt{3}}$ Or e.g. $\left(0, \dfrac{4}{3}\sqrt{3}\right]$	Correct range of y or $f(x)$. Also allow ft on their maximum **exact** value if both of the M's have been scored. Allow f or "range" for $f(x)$.	A1ft
			[5]
(c)	The function f is many-one	Also accept "the function f is not one-one" or "the inverse is one-many". **This mark should be withheld if there are contradictory statements.**	B1
			[1]
			12

		Question 14 Notes
14 (c)	**Note**	Accept • f is many to one (or 2 values in domain of f map to one in the range) • f is not one to one • f^{-1} would be one to many • the inverse would be one to many • it would be one to many • it is not one to one • the graph illustrates a many to one function Do NOT allow • it is many to one • You can't reflect in $y = x$ **Any reference to "it" we must assume refers to the inverse because of the wording in the question**